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The quadratic spin squeezing operators

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Abstract
We discuss generic spin squeezing operators (quadratic in angular momentum operators) capable of squeezing out quantum-mechanical noise from a system of two-level atoms (spins) in a coherent state. Such systems have been considered by Kitagawa and Ueda (1991 *Phys. Rev. Lett.* **67** 1852, 1993 *Phys. Rev.* A **47** 5138) in this context and a Hamiltonian of this nature governs the Lipkin model (Lipkin *et al* 1965 *Nucl. Phys.* **62** 188) which is relevant to nuclear physics.

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1. Introduction

We consider a system of N two-level atoms (TLA) and represent them by the algebra of spin one-half particles with total angular momentum $\mathbf{J}$ with $j = N/2$. Hence, we refer to the system as a system of N spins or TLA. We start with the system in a coherent state [1]

$$\begin{aligned}|j,\xi\rangle &= \mathrm{e}^{\xi J_- - \xi^* J_+}|j, m = +j\rangle \\ &= \frac{1}{(1+|\xi|^2)^j}\sum_{n=0}^{2j}\sqrt{{}^{2j}C_n}\,\xi^n|j, m = j-n\rangle \end{aligned} \tag{1}$$

where ξ is a complex number given by

$$\xi = \tan(\theta/2)\,\mathrm{e}^{\mathrm{i}\phi}.$$

$\theta = 0$ represents the state with all the atoms in the upper of the two states (spin-up). The coherent state satisfies the equality sign in the uncertainty relation for the components of 'angular momentum' in a plane normal to the mean-spin vector $\langle\mathbf{J}\rangle$ with equal amount of noise in its two quadratures. The mean-spin vector has the magnitude

$$|\langle\mathbf{J}\rangle| = \sqrt{\langle J_x\rangle^2 + \langle J_y\rangle^2 + \langle J_z\rangle^2}. \tag{2}$$

In other words, in a frame in which $\langle \mathbf{J} \rangle$ is along an axis (z' say), the minimum uncertainty condition is satisfied by the state described by equation (1), namely,

$$\Delta J_{x'} = \Delta J_{y'} = \sqrt{|\langle \mathbf{J} \rangle|/2} \tag{3}$$

where $\Delta J_{x',y'} = \sqrt{\left\langle J_{x',y'}^2 \right\rangle - \langle J_{x',y'} \rangle^2}$ are the variances in the two quadratures. If in such a frame the condition

$$\Delta J_{x'} \sqrt{2/|\langle \mathbf{J} \rangle|} \qquad \text{or} \qquad \Delta J_{y'} \sqrt{2/|\langle \mathbf{J} \rangle|} < 1 \tag{4}$$

is satisfied in some state of the system, then we shall say that the atomic system is spin squeezed.

Model Hamiltonians through which spin squeezed states could be realized were proposed by Kitagawa and Ueda [2] and others [3]. Since then, it has been vigorously studied in various atom–field systems [4–12]. When a TLA interacts with a squeezed radiation field, squeezing can be transferred from the field to the atomic system [4]. Self-squeezing of the spin system is possible if it is adequately nonlinear in its interaction with the radiation field [5]. The well-known Tavis–Cummings model also exhibits spin squeezing [6]. Its importance has been enhanced further due to its potential application in the area of quantum information [7–10].

We have been studying various quantum-mechanical operators, rather nonlinear in spin operators, capable of producing spin squeezed states when applied to a TLA. In [11], spin squeezing properties of a pseudo-Hermitian operator

$$\Lambda = (\mathrm{e}^{\epsilon} J_+ + \mathrm{e}^{-\epsilon} J_-)/2 \tag{5}$$

with real ϵ have been studied. The operator Λ has the features of pseudo-Hermicity as $\Lambda^\dagger \neq \Lambda$ but yet $\Lambda^\dagger = \eta \Lambda \eta^{-1}$ where η is a linear invertible Hermitian operator. Further, η can be written as $\eta = O^\dagger O$ where $O = O^\dagger = \exp(-\epsilon J_z)$ and, so, the eigenstates of Λ have real eigenvalues. This operator is of interest in the study of a TLA interacting with a squeezed vacuum [4]. Studies in [2, 12] reveal that with a Hamiltonian quadratic in the spin operator J_z, that is,

$$\hat{H}_1 = -\Gamma J_z^2 \tag{6}$$

(where Γ is a constant characteristic of the system) the corresponding evolution operator $\exp\left(-\mathrm{i}\Gamma t J_z^2\right)$ when applied to a spin coherent state in equation (1) produces spin squeezing in the TLA. This operator can describe the interaction of a TLA with the radiation field in a high-Q dispersive cavity [13]. This operator is also a special case of the so-called Lipkin–Meshkov–Glick (LMG) Hamiltonian for a many-body fermionic system [14].

In this paper, we identify the quadratic atomic (spin) operator which can introduce a squeeze from an analogy between the spin and bosonic systems. We do so in section 2 and study its squeezing properties in the following sections. In section 3, we give analytical results for a two-atom system which provide some insight into the properties of the proposed operator. The numerical results for $N > 2$ are discussed in section 4. We conclude the paper in section 5 with a comment on possible realization of such operators in some physical systems.

2. The operator

We begin with the derivation of a displacement operator for the spin coherent state in equation (1). This can be achieved if we cast the state $|j, \xi\rangle$ in the Schwinger representation [15]. The spin operators in the Schwinger representation are constructed by defining bosonic annihilation operators for two modes $a_i (i = +, -)$ such that $\left[a_i, a_j^\dagger\right] = \delta_{ij}$ and $[a_i, a_j] = 0 = \left[a_i^\dagger, a_j^\dagger\right]$. They take the forms $J_+ = a_+^\dagger a_-$, $J_z = \left[a_+^\dagger a_+ - a_-^\dagger a_-\right]/2$, etc. The spin coherent state $|j,\xi\rangle$ takes the form

$$|j,\xi\rangle = \frac{1}{(1+|\xi|^2)^j\sqrt{2j!}}\sum_{n=0}^{2j}{}^{2j}C_n\xi^n(a_+^\dagger)^n\left(a_-^\dagger\right)^{2j-n}|0_+,0_-\rangle$$
$$= D_{+,-}|0_+,0_-\rangle \tag{7}$$

where $|0_+,0_-\rangle$ are vacuum (fictitious) states for + and − modes. Comparing this form of the spin coherent state with the Glauber–Sudarshan coherent state for bosonic particles [16]

$$|\alpha\rangle = D(\alpha)|0\rangle \tag{8}$$

where $D(\alpha)$ is the well-known displacement operator

$$D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$$
$$= \mathrm{e}^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n(a^\dagger)^n}{n!} \tag{9}$$

it may be said that $D_{+,-}$ via the Schwinger construction is a 'displacement' operator for spin systems.

The above analogy between Glauber–Sudarshan coherent states and the spin coherent states suggests that one may attempt to obtain spin squeezed states in a manner similar to that employed for the bosonic case. Thus a quadratic form of the generators a and $a^\dagger$ of the Heisenberg–Weyl algebra was used to set up the operators which when acting on the Glauber–Sudarshan coherent states resulted in the squeezed states, namely, taking a cue from [17],

$$S_{\text{bosonic}} = \exp\left[\tfrac{1}{2}(\chi^* a^2 - \chi a^{\dagger 2})\right]. \tag{10}$$

Likewise, for the atomic or (pseudo)-spin coherent states it is tempting to consider exponentiation of the quadratic form constructed out of the generators J_x, J_y and J_z of the rotation group. Consider such a most general form, namely, J_kJ_l where $l.k$ run from x, y, z. Note that J_l transforms like a vector operator (in the three-dimentional representation) under rotation and accordingly J_lJ_k is a tensor of the second rank. However, this may be reduced in the following manner:

$$J_lJ_k = \left[\tfrac{1}{2}(J_lJ_k + J_kJ_l) - \tfrac{1}{3}\delta_{lk}J^2\right] + \left[\left(\tfrac{1}{2}(J_lJ_k - J_kJ_l)\right)\right] + \left[\tfrac{1}{3}\delta_{lk}J^2\right].$$

The first term in square brackets is the components of a traceless symmetric second-rank tensor (with five independent components), the second is the components of the antisymmetric second-rank tensor which by virtue of the commutation relation $[J_k, J_l] = \mathrm{i}\epsilon_{lks}J_s$ is expressed in terms of the vector representation and the last term in square backets is a scalar J^2. This is tantamount to the reduction of the direct product of two vectors

$$3\otimes 3 = 5\oplus 3\oplus 1.$$

Thus the exponentiation in $\exp \mathrm{i}\eta J_kJ_l$ being quadratic in the generation of the rotation group can be decomposed into the sum of three operators with the trace $\sum J_kJ_l\delta_{kl} = J^2$ yielding a mere overall phase, the antisymmetric form in the exponent just a rotation, and the quadratic form in an exponent yielding the non-trivial squeeze. Since the different components of the quadrupole tensor are related via rotations we can in effect express the squeezing operator in the case of the spin system choosing, for example, the fiducial quadratic forms

$$S_{\text{spin}} = \exp[\eta(J_xJ_y + J_yJ_x)]$$
$$S'_{\text{spin}} = \exp\left[\eta J_z^2\right]. \tag{11}$$

Thus $S_{\rm spin}$ can represent the time evolution of a state vector under the action of a Hamiltonian

$$H = \zeta[J_x J_y + J_y J_x] \tag{12}$$

where $\eta = -{\rm i}\zeta t$.

Indeed, H is connected to other quadratic combinations of spin operators, namely, $[J_y J_z + J_z J_y]$ and $[J_x J_z + J_z J_x]$ by mere rotations. Similarly, $\hat{H}_1$ in equation (6); ${J_x}^2$ and ${J_y}^2$ are related to one another by simple rotations. Thus the operators in equation (11) can be taken to be the representatives of interaction Hamiltonians having quadratic forms of spin operators. Kitagawa and Ueda [2] has shown that the operator $S'_{\rm spin}$ in equation (11) produces spin squeezing resulting from a single-axis twisting and the operator $S_{\rm spin}$ in equation (11) or the Hamiltonian in equation (12) can give rise to spin squeezing resulting from two-axis counter twisting. We, however, present a detailed account of this operator and show that a large number of atoms (spins) can be squeezed. We also point out the possibilities of its physical realization.

In [12] we have already studied the spin squeezing dynamics of $S'_{\rm spin}$. What follows is a discussion on spin squeezing properties of the operator.

We can write J_+ and J_- in the Schwinger representation and proceed to examine the spin squeezing properties when operated upon a spin coherent state. This may produce a spin squeezed state

$$|sss\rangle = S_{\rm spin} D_{+,-}|0_+, 0_-\rangle$$

in the lines of Yuen's representation for a bosonic squeezed state [17]. However, this method gets very complicated and, so, we investigate the squeezing properties of $S_{\rm spin}$ as follows.

We assume that the operator acts on a system of atoms prepared initially in a coherent state $|\theta,\phi\rangle \equiv |j,\xi\rangle$. This can be achieved by sending the atoms through a cavity whose single-mode is maintained by the radiation from a laser. The polar angle θ is decided by the interaction time of the atoms with the cavity field. We first consider the action of $S_{\rm spin}$ on a two-atom coherent state as it gives analytical results. With the insight gained from these results, we go ahead with the numerical analysis of squeezing properties of $S_{\rm spin}$ acting on a system having more than two atoms.

3. A two-atom system

In addition to this being the first step of our general study, the bipartite system is of special interest in the study of quantum entanglement [7–10].

The initial state of the system, the coherent state, is in this case given by

$$|\xi,\theta,\phi\rangle = \frac{1}{(1+|\xi|^2)}[|1,+1\rangle + \sqrt{2}\xi|1,0\rangle + \xi^2|1,-1\rangle] \tag{13}$$

which is a linear superposition of the three Wigner states with $m = 0, \pm 1$. The action of $S_{\rm spin}$ on the $|\xi,\theta,\phi\rangle$ changes the amplitudes of the Wigner states giving

$$\begin{aligned}|sss\rangle &= S_{\rm spin}|\xi,\theta,\phi\rangle\\ &= C_1|1,+1\rangle + C_2|1,0\rangle + C_3|1,-1\rangle\end{aligned} \tag{14}$$

where

$$C_1 = \frac{1}{(1+|\xi|^2)}[\cos(2|\eta|) + \sqrt{\eta/\eta^*}\xi^2\sin(2|\eta|)],$$

$$C_2 = \frac{\sqrt{2}\xi}{(1+|\xi|^2)}$$

and

$$C_3 = \frac{1}{(1+|\xi|^2)}[\xi^2\cos(2|\eta|) - \sqrt{\eta^*/\eta}\sin(2|\eta|)].$$

It has been shown in [18] that this state radiates squeezed light [17], thus indicating strongly that the collective two-atom state is spin squeezed [5]. However, its spin squeezing properties, as is known by now [2–12] and defined in equation (4), have not been discussed there. We present them below.

It is straightforward to calculate the moments and correlation functions of the spin operators. We find that all the three correlation functions, namely $\langle J_xJ_y + J_yJ_x\rangle$, $\langle J_xJ_z + J_zJ_x\rangle$ and $\langle J_yJ_z + J_zJ_y\rangle$ are nonzero and for real ξ and η one of them, $\langle J_xJ_z + J_zJ_x\rangle$, survives. This indicates the squeezing capabilities of $S_{\rm spin}$ for all possible values of ξ and η. This is due to the fact that nonzero values of the correlation function (at least one) are a necessity for spin squeezing to take place [2, 11]. The variances in the two quadratures, namely x' and y', in the rotated frame as defined in equations (2)–(4) are given by

$$\begin{aligned}(\Delta J_{x'})^2 = \frac{1}{|\langle\mathbf{J}\rangle|^2}\Bigg[&\frac{1}{2(1+\xi^2)^4}\{(\xi^8-2\xi^4+1)\cos^2(4\eta) - 2\xi^2(\xi^4-1)\sin(8\eta) + 4\xi^4\sin^2(4\eta)\}\\
&+\frac{1}{(1+\xi^2)^6}\Bigg\{(3\xi^{10}-10\xi^6+3\xi^2)\cos^3(4\eta) - 3(\xi^{10}-2\xi^6+\xi^2)\cos^2(4\eta)\\
&-2(\xi^{10}-4\xi^8-4\xi^6-4\xi^4+\xi^2)\cos(4\eta)\\
&-\frac{1}{2}(\xi^{12}-15\xi^8+15\xi^4-1)\sin^3(4\eta) - 12\xi^6\sin^2(4\eta)\\
&+\frac{1}{2}(\xi^{12}+8\xi^{10}-11\xi^8+11\xi^4-8\xi^2-1)\sin(4\eta)+6\xi^4\\
&\times(\xi^4-1)\sin(8\eta)+4\xi^{10}+8\xi^6+4\xi^2\Bigg\}\Bigg]\end{aligned} \tag{15}$$

and

$$(\Delta J_{y'})^2 = \frac{1}{2}\left[1+\frac{1}{(1+\xi^2)^2}\{2\xi^2(1-\cos(4\eta)) + (1-\xi^4)\sin(4\eta)\}\right]. \tag{16}$$

In equation (15), the presence of $|\langle\mathbf{J}\rangle|^2$ in the denominator is due to the rotation required to bring $\langle\mathbf{J}\rangle$ along the required direction, here, z'-axis. The mean-spin vector is given by

$$\begin{aligned}|\langle\mathbf{J}\rangle|^2 = \frac{1}{(1+\xi^2)^4}[&4\xi^2\{1+\xi^4+2\xi^2\cos(4\eta) - (1-\xi^4)\sin(4\eta)\}\\
&+(1-\xi^4)^2\cos^2(4\eta) + 4\xi^4\sin^2(4\eta) - 2\xi^2(\xi^4-1)\sin(8\eta)].\end{aligned} \tag{17}$$

It is easy to note that for $\eta = 0$ the results reduce to that for a spin coherent state, that is, $|\langle\mathbf{J}\rangle| = 1$ and $\Delta J_{x'} = \Delta J_{y'} = 1/\sqrt{2}$ in accordance with equations (2) and (3) for a two-atom system. Further, we note in equations (15) and (16) that the arguments of all the circular functions appearing there are in the form of 4η and 8η. Hence, the variances return to their initial condition, that is a coherent state, for $\eta = n\pi/2$ where n is an integer. The spin squeezing parameters $S_{x'}$ and $S_{y'}$ as defined in equation (4)

$$S_{x',y'} = \Delta J_{x',y'}\sqrt{2/|\langle\mathbf{J}\rangle|} \tag{18}$$

have interesting relations for $\theta = 0$ and π:

$$S_{x'}|_{\theta=0} = S_{y'}|_{\theta=\pi} = \sqrt{\frac{1+\sin(4\eta)}{\cos(4\eta)}} \tag{19}$$

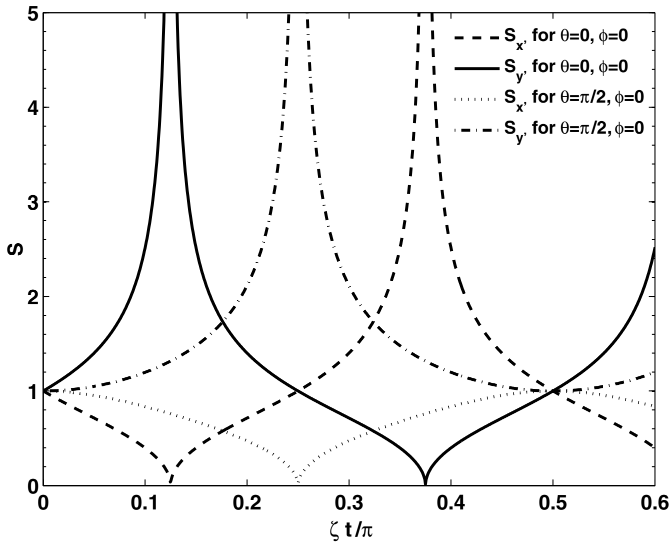

Figure 1. Squeezing S in the two-atom system is plotted versus $\eta = \zeta t$ in units of π. Note that for $\theta = \pi$, curves are the same as for $\theta = 0$ but $S_{x'}$ and $S_{y'}$ are interchanged.

and

$$S_{x'}|_{\theta=\pi} = S_{y'}|_{\theta=0} = \sqrt{\frac{1 - \sin(4\eta)}{\cos(4\eta)}}. \tag{20}$$

Because of the above observations it is sufficient to study the variances for $0 < \eta \leqslant \pi/2$. We display them in figure 1 and note perfect squeezing of the two-spin system.

4. Systems with $j > 1$

For a system having more than two two-level atoms, it is highly complicated to get an analytical expression. However, the problem can be tackled somewhat numerically.

We are concerned about the evolution

$$|sss\rangle = S_{\mathrm{spin}}|j, \xi\rangle \tag{21}$$

where $j > 1$. Quantum-mechanical average of any spin operator or products of them, say $\hat{O}$, is given by

$$\langle\hat{O}\rangle = \langle sss|\hat{O}|sss\rangle = \sum_{m'=-j}^{j}\sum_{m''=-j}^{j}\langle sss|j, m'\rangle\langle j, m'|\hat{O}|, m''\rangle\langle j, m''|sss\rangle. \tag{22}$$

The inner product $\langle j, m|sss\rangle$ is given by

$$\begin{aligned}\langle j, m|sss\rangle &= \langle j, m|S_{\mathrm{spin}}|j, \xi\rangle \\ &= \sum_k \langle j, m|sss_k\rangle\langle sss_k|j, \xi\rangle\, \mathrm{e}^{-\mathrm{i}\lambda_k t}\end{aligned} \tag{23}$$

where we have used $S_{\mathrm{spin}} = \exp(-\mathrm{i}Ht)$ with H being given by equation (12). The $|sss_k\rangle$ are the eigenvectors of H with eigenvalues λ_k.

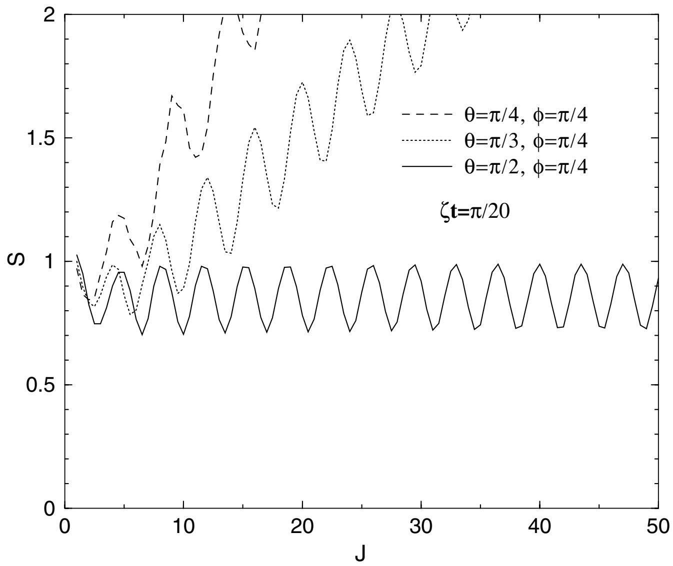

Figure 2. Squeezing in y' quadrature as a function of j. Note that there is no squeezing in the other quadrature for such interaction times.

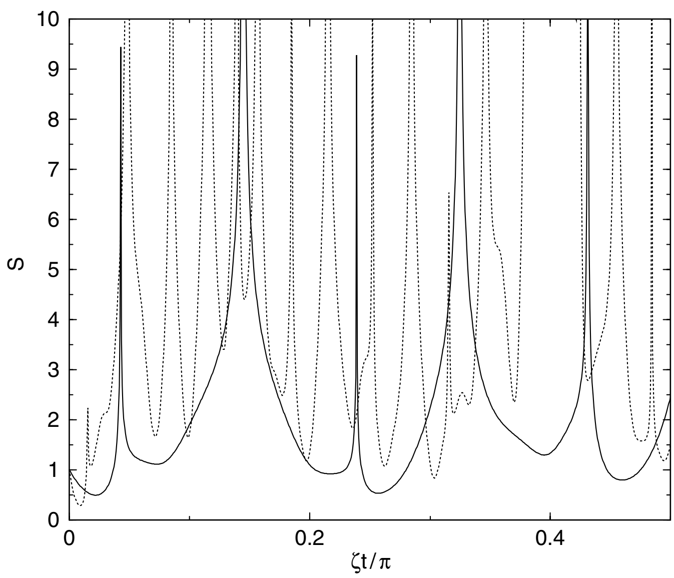

Figure 3. Oscillations in squeezing in x' quadrature as a function of $\eta = \zeta t$. $\theta = \phi = 0$. The solid and dotted lines are for $j = 4$ and 15 respectively.

The dynamics exhibit a rich variety of characteristics of spin squeezing. In figure 2, we display S'_y for short-interaction time, that is, $\eta = \zeta t = \pi/20$. The striking feature there is that, for $\theta = \pi/2$, S'_y oscillates just below the line $S'_y = 1$ indicating spin squeezing for all values of j. Thus, the operator S_{spin} is capable of squeezing a large number of atoms if they are initially prepared in a coherent state such that $\langle j, \xi|J_z|j, \xi\rangle = 0$. The distribution function for the initial condition $\theta = \pi/2$ takes the form

$$P(j, m) = |\langle j, m|j, \xi = 1\rangle|^2 = \frac{1}{2^{2j}} \frac{(2j)!}{(j+m)!(j-m)!} \tag{24}$$

where we have taken $\phi = 0$ for simplicity. It can easily be shown that $P(j, m)$ peaks at $m = 0$ by using the expression for the digamma function, the derivative of the factorial function. The state $|j, m = 0\rangle$ has the property of maximum correlation among individual spins [19], which is exploited by the operation $S_{\rm spin}$ to squeeze out noise displayed in figure 2. This type of behavior has also been noticed in the spin squeezing properties of pseuo-Hermitian operator discussed in [11]. This striking property of the spin squeezing operator persists for longer interaction times too.

We display in figure 3 the variations in squeezing as a function of time for $j > 1$. A comparison with figure 1 indicates that the oscillations in squeezing with time increase with j.

5. Conclusions

We have shown that operators in the form of exponentian of quadratic forms of spin operators, $J_k J_l$, have the spin squeezing properties when operated upon a spin coherent state. We have shown in figure 2 that the operator can squeeze out noise from an ensemble having a large number of spins initially prepared in a coherent state with $\theta = \pi/2$ and $\phi = \pi/4$. As shown there, a deviation from this initial condition reduces the number of squeezed spins drastically. These results, however, would be influenced by the surrounding reservoirs which we plan to study in a later publication.

The quadratic forms of spin operators have been discussed in the literature for quite sometime. For example, a widely discussed interaction is the so-called Lipkin interaction Hamiltonian [14]

$$H = G_1\left(J_+^2 + J_-^2\right) + G_2(J_+J_- + J_-J_+), \tag{25}$$

where G_1 and G_2 are the coupling constants representing the two-body interactions. Indeed these two types of terms are the archetypes of the Hamiltonian quadratic in the generators and related to what we expressed through equations (11). The operators in equations (11) are special cases of the Lipkin Hamiltonian. The $S_{\rm spin}$ also appears in the Hamiltonian of a complex magnetic molecule in a static magnetic field [20]. The operator $S'_{\rm spin}$ can be realized in an optical system consisting of an ensemble of atoms in a high-Q dispersive cavity [13]. We have studied its behavior in detail in [12]. Also, the exponent in $S'_{\rm spin}$ describes atomic cooperation in a Dicke system [19] placed inside a cavity and driven by an external laser field [21]. Hence, its Hamiltonian includes a term which is proportional to J_z^2. Our recent study shows that this system exhibits spin squeezing [22].

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References

[1] Radcliffe J M 1971 *J. Phys. A: Gen. Phys.* **4** 313
Arecchi F T, Courtens E, Gilmore R and Thomas H 1972 *Phys. Rev.* A **6** 2211

[2] Kitagawa M and Ueda M 1991 *Phys. Rev. Lett.* **67** 1852
Kitagawa M and Ueda M 1993 *Phys. Rev.* A **47** 5138
[3] Wineland D J, Bollinger J J, Itano W M, Moore F L and Heinzen D J 1992 *Phys. Rev.* A **46** R6797
Wineland D J, Bolinger J J, Itano W M and Heinzen D J 1992 *Phys. Rev.* A **50** 67
[4] Agarwal G S and Puri R R 1994 *Phys. Rev.* A **49** 4968
[5] Vernac L, Pinard M and Giacobino E 2000 *Phys. Rev.* A **62** 063812
Dantan A, Pinard M, Josse V, Nayak N and Berman P 2003 *Phys. Rev.* A **67** 045801
Dantan A and Pinard M 2004 *Phys. Rev.* A **69** 043810
[6] Genes C, Berman P and Rojo A G 2003 *Phys. Rev.* A **68** 043809
[7] See, for example, Hald J, Sørensen L, Schori C and Polzik E S 1999 *Phys. Rev. Lett.* **83** 1319
Kuzmich A, Mandel L and Bigelow N P 2000 *Phys. Rev. Lett.* **85** 1594
Meyer V, Rowe M A, Kielpinski D, Sackett C A, Itano W M, Monroe C and Wineland D J 2001 *Phys. Rev. Lett.* **86** 5870
[8] Mølmer K and Sørensen A 1999 *Phys. Rev. Lett.* **82** 1835
Sørensen A and Mølmer K 1999 *Phys. Rev. Lett.* **82** 1971
Unanyan R G and Fleischhauer M 2003 *Phys. Rev. Lett.* **90** 144601
[9] Sackett C A *et al* 2000 *Nature (London)* **404** 256
[10] Sørensen A, Duan L-M, Cirac J I and Zoller P 2001 *Nature (London)* **409** 63
[11] Deb R N, Nayak N and Dutta-Roy B 2005 *Eur. Phys. J.* D **33** 149
Nayak N, Deb R N and Dutta-Roy B 2005 *J. Opt. B: Quantum Semiclass. Opt.* **7** S761
[12] Deb R N, Sebawe Abdalla M, Hassan S S and Nayak N 2006 *Phys. Rev.* A **73** 053817
[13] Agarwal G S, Puri R R and Singh R P 1997 *Phys. Rev.* A **56** 2249
[14] Lipkin H J, Meshkov N and Glick A J 1965 *Nucl. Phys.* **62** 188
[15] Rose M E 1957 *Elementary Theory of Angular Momentum* (New York: Dover)
[16] Glauber R J 1963 *Phys. Rev. Lett.* **10** 84
Sudarshan E C G 1963 *Phys. Rev. Lett.* **10** 277
[17] Stoler D 1970 *Phys. Rev.* D **1** 3217
Yuen H P 1976 *Phys. Rev.* A **13** 2226
Caves C M 1981 *Phys. Rev.* D **23** 1693
[18] Barnett S M and Dupertuis M-A 1987 *J. Opt. Soc. Am.* B **4** 505
Dupertuis M-A, Barnett S M and Stenholm S 1987 *J. Opt. Soc. Am.* B **4** 1124
[19] Dicke R H 1954 *Phys. Rev.* **93** 99
[20] Wernsdorfer W and Sessoli R 1999 *Science* **284** 133
Keçecioğlu E and Garg A 2001 *Phys. Rev.* B **63** 064422
[21] Hassan S S and Bullough R K 1981 *Optical Bistability* ed C M Bowden, M Ciftan and H R Robl (New York: Plenum) p 367
[22] Hassan S S, Nayak N and Deb R N 2008 (in preparation)